Congruent Numbers Revisited: Modular Patterns and Arithmetic Classification in Four Families

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Abstract. We investigate four algebraic families that generate congruent numbers, each exhibiting distinct modular residue pattern modulo 8. We prove that these modular constraints persist at at the level of their square-free parts, linked through the existence of rational points on corresponding elliptic curves. We provide a refined arithmetic classification of congruent numbers within these parametrized families.

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